Intrinsic, damped, and forced spring oscillations (remote experiment)

Tasks:

1. Find out from several measurements the intrinsic frequency of the damped oscillator $\omega_b = (\omega_o^2 - b^2)^{1/2}$, and also the damping b.
2. Plot the dependence of the amplitude of the driven oscillations on the angular frequency of the driving force ω_v. Determine the resonance frequency $\omega_{v,\,res}$.
3. Try to evaluate for several frequencies of the driving force ω_v the initial phase of forced oscillations φ_v. Plot the graph.

Remote experiment available at http://ises3.prf.ujep.cz/index_en.html.

Measurement principle

Harmonic oscillator

A body making an oscillatory harmonic motion is called a harmonic oscillator. At the first approximation, it may be, e.g. a body suspended on a spring (Fig. 1) or an atom or a molecule of a solid.

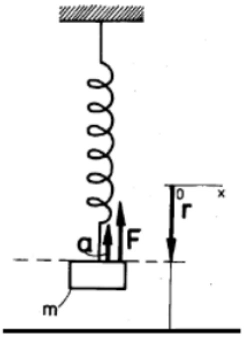

Figure 1. The schematic representation of the harmonic oscillator; *F* is the elastic force, *r* is the deflection, *a* is the acceleration

If the damping resistance of the environment may be neglected, we speak of the undamped harmonic oscillator. We can demonstrate that the oscillation is harmonic if the acting force is proportional to the deflection from the equilibrium and its direction is opposite to that of the deflection.

Let us have a spring, for the deflection of the equilibrium position by r is necessary to exert the force $F = -kr$, where k is the stiffness of the spring. An equation of the motion of the harmonic oscillator, consisting of the spring and the weight with the mass m, is $-kr = ma$, that is,

$$-k\mathbf{r} = m\frac{d^2\mathbf{r}}{dt^2} \qquad (1)$$

Introducing $\omega_0 = (k/m)^{1/2}$ this equation can be expressed as follows

$$\frac{d^2\mathbf{r}}{dt^2} + -\omega_0\mathbf{r} = 0 \qquad (2)$$

A general solution of this equation in the one-dimensional case in the y axis ($r = y$j)

$$y = A \sin(\omega_0 t + \varphi), \qquad (3)$$

where y is the deflection, A is the amplitude, ω_o is the intinsic angular frequency of the harmonic oscillator, and φ is its initial phase (Fig. 2).

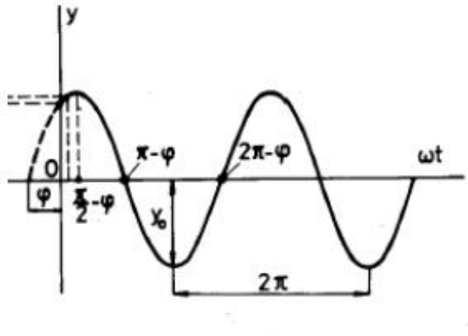

Figure 2. Graphical representation of the harmonic motion

We can write for the natural angular frequency of the undamped harmonic oscillator

$$\omega_0 = \left(\frac{k}{m}\right)^{\frac{1}{2}} = 2\pi f_0 = 2\pi \frac{1}{T_0}, \qquad (4)$$

here f_o is its natural frequency and T_o is the natural period of the harmonic oscillator.

Damped oscillator

During the motion of a harmonic oscillator under real-world conditions, friction forces are always acting, decreasing the amplitude of the oscillating motion, leading to a standstill after some time. This kind of oscillation is then referred to as a damped oscillator.

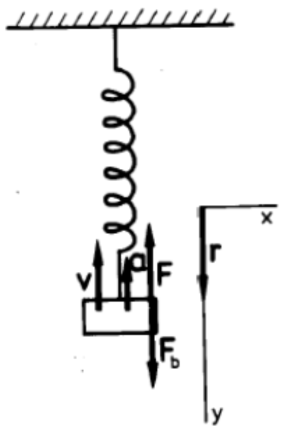

Figure 3. Schematic representation of the damped oscillator; F is the elastic force, F_b is the damping force, r is the deflection, v is the velocity and a is the acceleration

The friction force at low velocities is directly proportional to the instantaneous velocity $\boldsymbol{v}$ of the weight

$$\boldsymbol{F}_b = -k_b\boldsymbol{v} = -k_b\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} \qquad (5)$$

or

$$-k\boldsymbol{r} - k_b\boldsymbol{v} = m\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2} \qquad (6)$$

Let us denote the quantity $b = k_b/(2m)$, referred to as the damping, $[b] = \mathrm{s}^{-1}$. For a small damping ($b < \omega_o$) equation (5) has a solution for $r = y$j

$$y = A\mathrm{e}^{-bt}\sin(\omega_b t + \varphi) \qquad (7)$$

where $\omega_b = (\omega_o^2 - b^2)^{1/2}$ is the angular frequency of the damped oscillator.

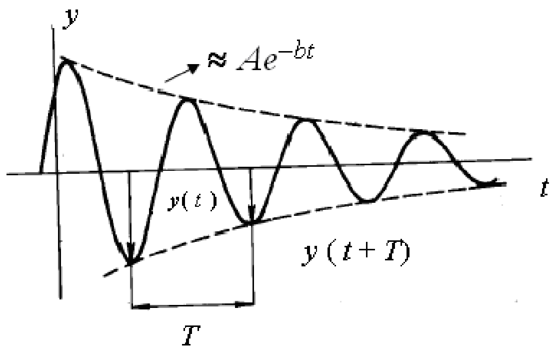

Figure 4. Graphical representation of the damped oscillator

Driven oscillator

The important case of the ocillating motion is the driven oscillator, where an external force causes a material object to oscillate at a frequency generally different from that of the natural frequency of the oscillator. If, however, both frequencies are approaching each other, the important phenomenon that results is known as resonance.

Let us assume that the weight in Fig. 3 is acted upon by a driving periodic force

$$F_v = F_0\sin(\omega_v t) \qquad (8)$$

where F_o is the amplitude and ω_v the angular frequency of the driving force F_v.

If all the forces that cause the weight to move are considered, the following equation of motion may be written

$$-ky - k_b\frac{\mathrm{d}y}{\mathrm{d}t} + F_0\sin(\omega_v t) = m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} \qquad (9)$$

and after some adjustment

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega_0^{\ 2} y = F_0 \sin(\omega_v t) \qquad (10)$$

The solution of this equation then is

$$y = A_v \sin(\omega_v + \varphi_v) \qquad (11)$$

where A_v and φ_v are the amplitude and the initial phase of the driven oscillations, fulfilling

$$A_v(\omega_v) = \frac{F_0}{m\left[\left(\omega_0^{\ 2} - \omega_v^{\ 2}\right)^2 + 4b^2\omega_v^{\ 2}\right]^{1/2}} \qquad (12)$$

$$\mathrm{tg}\,\varphi_v(\omega_v) = -\frac{2\omega_v b}{\omega_0^{\ 2} - \omega_v^{\ 2}} \qquad (13)$$

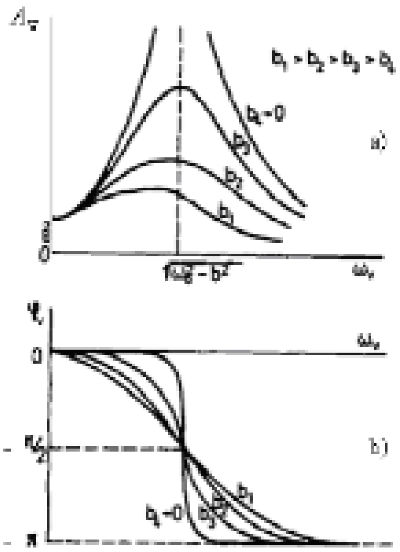

Figure 6. Graphical representation of the driven oscillations on the driving force frequency ω_v with damping b as a parameter; the amplitude A_v (part a) and initial phase φ_v (part b)

The graphs in Fig. 6 reveal some interesting features of the driven oscillations. First, it is obvious that the amplitude of the driven oscillations A_v (see Fig. 6 a) is a function of the angular frequency of the driving force ω_v. The maximum amplitude of the driven oscillations occurs for the so called resonance frequency, $\omega_{v,\ res}$

$$\omega_{v\,rez} = (\omega_0^{\ 2} - b^2)^{1/2} \qquad (14)$$

with the value $A_{v,res} = F_o/(2bm(\omega_o^2 - b^2)^{1/2}$. Similarly, the initial phase φ_v of the driven oscillations (or the initial phase shift between the driving force F_v and the deflection y) depends on the frequency of the driving force ω_v and at resonance, equals $\varphi_{v,res} = -\pi/2$ (see Fig. 6 b).

Experiment description

The aim of a remote experiment is to allow the experimentator to observe and measure the damped and driven oscillation of the spring and to explain all its properties, interesting facts, and characteristic physical quantities. The use of this experiment is wide: determination of the the angular frequency of harmonic oscillator, damping ratio, resonance, phase, and amplitude, and transformation of energy from the source of driven force on the oscillator. The online web camera provides an online real-time view of the experiment. The camera has two resolutions and two switches to turn on/off. Higher resolution increases the amount of data transferred.

Measurement procedure

1. Familiarise yourself with the experiment, the damped oscillations, and the forced oscillations of the oscillator. Shortly after pressing the button "Run experiment" there will appear a page with experiment. Please be patient before connecting to the experiment and to all data loading. If someone is measuring at the moment and the experiment aparature is occupied, wait 5 minutes. The reservation system is ready and will run in a few seconds.
2. The Web page allows control of the experiment by changing frequency of the electromagnetic generator, which produces an outer periodic driven force that affects body mass *m*. Set the frequency. There are two possibilities you can choose from set of predefined frequencies by clicking on buttons with their values: 1.0, 1.2, 1.3, 1.4 Hz and the stop button by using slider (continuous change) in allowed interval.
3. Observe the time dependence of the driven force for various frequencies (red curve).
4. Observe time dependence of instantaneous position for various frequencies of driven force (green curve).
5. Measure the damped oscillations of the oscillator. This can be done by turning off the driving force and monitoring the oscillations of free oscillations. Save your data.
6. Measure forced oscillations for the entire frequency interval of the driving force ω_v, save your data.
7. Data record: You have the possibility to record the data and save them. To do that, press the button "Start recording", after you finish your measurement, press "Stop recording". Then you are able to see both measured time dependences: driven force, instantaneous position, bottom window. You can also download data measured by other experimentators.
8. Export of measurement data to PC: Drop down the recorded measurement list, choose the number of experiments which you wish to export, and choose the data format (comma-separated values, values for Excel). Press the button, and a new window with data in chosen format will appear (check whether you have allowed pop-ups for the experiment site). If you check the option Repeat experiment, you don't have to select numer of experiment (it will be the same for all) and measured data are in your PC. When the pop-up window appears, press Ctrl + A to select them all, Ctrl + C to copy them, and Ctrl + V to paste them into your programme (e.g. Excel). Paste the data in a table and create a chart. Data are sorted into four columns: time, excitation current, driven force, and instantaneous position.
9. From measured values we can create graph dependence of driven oscillation amplitude on driven force frequency using arithmetic means of amplitudes for certain frequencies of driven force. From there we are able to obtain the resonance frequency. The highest value of the amplitude corresponds to the resonance frequency of 1,333 Hz. When the frequency is the resonance frequency, the initial phase of the driven oscillations is $-\pi/2$.

The phenomenon of driven oscillations and resonance can be very often observed in nature and also in technology (in nature, e.g., in the absorption of UV radiation on ozone molecules; in technology, e.g., in resonance circuits in telecommunications). Think about the energy transferred by the driving force in the system with the oscillator. When is this energy transfer the greatest?